DISCUSSIONS AND CONTRIBUTIONS

Derivation of a Basic Efficiency Formula for Concentrating Operations

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Report of discussion at November, 1962, General Meeting (Chairman: Mr. J. B. Simpson, President). Paper published in September, 1962 (Transactions, *vol. 71, 1961–62*), *pp. 697–704*

Mr. E. Douglas, presenting his paper, first apologized for a considerable error which appeared in the second formula of the synopsis (p. 697). The first term of the denominator was printed as $100c - f$; that should have read $100c - Rf$. The same modification should be made on page 700.

Continuing, he said his paper, originally intended as part of the discussion to the paper by Stevens and Collins, published by the American Institute of Mining, Metallurgical and Petroleum Engineers, was consequently concerned to some extent with comparisons between the formula which those workers had recommended and the one described in the present paper as a basic formula for expressing concentration efficiency.

In deriving the formula, no attempt had been made to introduce economic biases nor had probabilities or intermediate rates of concentration been introduced. The sole concern had been, by examining the facts associated with the feed and with the products, to assess the effectiveness with which the constituents of a mixture had been separated.

For simplicity the concentrating operation had been considered in two stages—a simple weight split giving two products, in each of which the constituent ratios were identical and equal to those in the feed, followed by a simultaneous interchange of one constituent from one product with the other constituent from the other product.

A two-constituent feed was illustrated by the block diagram shown in Fig. 1 (p. 699). There the constituents had been designated in common terms as values and gangue, and the initial weight split was represented by the intermediate products AFGE and FBDG, each containing f per cent values. At that point no constituent separation had taken place and consequently the concentration efficiency would be rated as nil. During the second stage, values were moved from FBDG into the concentrate AFGE, the gangue moving in the opposite direction, with the resulting product grades of c per cent and t per cent. The efficiency with which the values had been transferred could be expressed as the weight of values which moved into the concentrate divided by the weight of values which were available for movement, i.e.

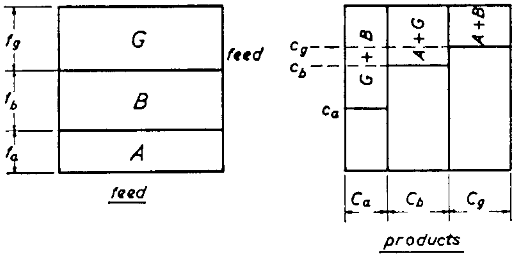
$$\frac{(c - f)\,C}{(100 - C)\,f} \times 100.$$

Similarly, the efficiency with which the gangue was rejected would be the weight of gangue rejected divided by the weight available for rejection, i.e.

$$\frac{(c-f)\,C}{(100-f)\,C} \times 100.$$

Thus it was possible to indicate the individual efficiencies of the two operations which influenced the concentration process. They could be combined in a number of ways, each of which would adequately represent the overall efficiency. For example, it could be expressed as the arithmetic mean or the geometric mean, or even more simply as the product of the two terms. The product combination had been used in deriving the formula presented in the paper.

Several variations could be applied to the general theme used in that derivation; for example, a slightly more logical, but considerably more complicated, representation could be had by introducing a factor expressing the rate of values concentration with respect to gangue rejection. An alternative but almost identical treatment could be applied to produce an efficiency formula to give directly the overall efficiency of an operation from which several concentrates were produced. One of the two block diagrams in Fig. A above represented the feed and the other the products of concentrations. Assuming the efficiency of concentration to be



$$E = \frac{C_a(c_a - f_a) + C_b(c_b - f_b) + C_g(c_g - f_g)}{(100 - C_a)\,f_a + (100 - C_b)\,f_b + (100 - C_g)\,f_g} \times 100$$

$$= \frac{\Sigma\ C\,(c-f)}{\Sigma\ f\,(100-C)} \times 100$$

Fig. A.

$$E = \frac{\text{Total weight of minerals concentrated with respect to a sampling quotation}}{\text{Total weight of minerals available for concentration with respect to a sampling operation}} \times 100$$

then

$$E = \frac{Ca\,(ca - fa) + Cb\,(cb - fb) + Cg\,(cg - fg)}{fa\,(100 - Ca) + fb\,(100 - Cb) + fg\,(100 - Cg)} \times 100$$

$$= \frac{\Sigma\ C\,(c-f)}{\Sigma\ f(100 - C)} \times 100 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For a two-constituent feed, or when the concentration of only one mineral of a multi-mineral feed was considered, the efficiency expression became

$$E = \frac{2C\,(c-f)}{f\,(100 - C) + C\,(100 - f)}$$

The appendix to the paper demonstrated the use of the efficiency formula by examples selected from independent practical operations. In one instance it had been used to determine the cut size in a cyclone operation—that had already aroused discussion.

He had attempted to devise a form of equation which was simple and which logically and consistently represented the concentration efficiency of any physical concentrating process. It gave a zero rating for a sampling operation and a maximum 100 per cent for the total separation of the constituents in a mixture.

Mr. T. H. Hughes* read the following contribution from Dr. A. J. Robinson.

Dr. A. J. Robinson: The importance of an efficiency formula which is acceptable to engineers and research workers can hardly be over-emphasized. At one time or another most engineers working in the field of mineral technology feel the need for an efficiency formula which is simple to use, which requires only that information which is normally obtained in test work and yet is based upon a sound theoretical concept. It is my belief that Mr. Douglas's formula meets these requirements. It is often useful to have a clear mental image of cause and effect when considering a unit operation but in some cases the use of an efficiency formula clouds the mental picture and may, in fact, make interpretation of performance difficult. Under these circumstances an efficiency formula does more harm than good.

However, there may be no alternative to the use of an efficiency formula for assessing the results of an *ad hoc* series of tests. This is particularly true in the field where there may be insufficient metallurgical staff to carry out carefully designed experiments because of the pressure of routine metallurgical control and accounting duties. Under these circumstances it is not uncommon for test work to be done on an 'as and when' basis and for tests to proceed from one good idea to another. Consequently, it is

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not always possible to determine precisely the effect of changing the operating variables; and an efficiency formula which is generally recognized offers a means of assessment which would be widely acceptable. Too early emphasis of economic considerations may mislead the investigator who attaches too much importance to either recovery or grade, and may mean that a promising route is abandoned. By using an efficiency formula incorporating both recovery and grade factors there is less chance of this happening.

In my own experience on difficult flotation problems, where selectivity is poor and simultaneous rejection of more than one constituent is attempted in one operation, to be followed by further operations for the recovery of value, the assessment of performance poses serious problems. By making assumptions that weight/recovery* curves should be straight lines, the assessment of the efficiency of various flotation procedures for the various constituents may be made using the slope of the respective recovery curves. Unfortunately this assessment is not necessarily true and the slope of the various weight recovery curves for the various minerals present cannot remain constant over the whole recovery range where collection occurs at different collector concentrations. In such cases the use of an efficiency formula can be extremely valuable and furthermore may save experimental time in amenability testing. The mathematician and research scientist would correctly recommend factorial design to truly assess the effects of variables, but before this may even be attempted some preliminary sorting of available routes, particularly in flotation, must be made; here the use of an efficiency formula is invaluable.

Another problem arises in the statistical analysis of factorial experiments; what is to be used to assess the results? Appropriate use of the efficiency formula which combines both 'recovery' and 'grade' provides a single measure of performance which lends itself to mathematical analysis.

Mr. J. R. Stevens said that the author suggested that concentration operations could be assessed by a fundamental formula. However, his derivation of such a 'fundamental and logically derived formula' was conspicuous by the absence of fundamental considerations. His formula was based on the concept of mass transfer across an imaginary boundary between the concentrate weight and the tailing weight; gangue transferring to the tailing and values of equal weight transferring to the concentrate. Such a process had no parallel in mineral processing and he found it difficult to appreciate the logic of such a concept. His interpretation of the author's concept was by inference, he admitted, but he did not think that any other interpretation could be made.

Reference had been made to a recent paper in which Mr. Collins and the speaker had defined new expressions for the assessment of mineral concentrating processes. There was another publication which antedated the references in the author's list and which might usefully be included—Harwell Research Group Report A.E.R.E. R2922, dated August, 1959*—that dealt with the same subject matter in more detail.

Referring to the author's six basic requirements (pp. 697–8), he still believed the second requirement to be valid—in fact the only modification he would suggest was that it should be underlined. It was because there was *no fundamental basis for any single quantifying expression* that the claim made by Mr. Douglas merited consideration. When referring to difficulty—requirement 5—it was not meant to imply that 'work in a closed system' should not be used as a basis for any formula, but that the practical difficulties, i.e. the 'external work', should not be included. If requirements 5 and 6 were met, i.e. the difficulty of attainment and the economic considerations were not included in a formula, then it should not be surprising if that efficiency formula gave results which were not always in accord with one's mental assessment of the efficiency of a given operation. The reason for such anomalies was undoubtedly due to the unconscious and inconsistent use of the yardsticks of difficulty and economics, when mentally comparing sets of results. *The concept of concentration efficiency was what determined the derived equation and there were as many possible formulae as there were definitions.* He felt that Mr. Douglas gave no adequate verbal basis for his efficiency expression.

The fact that the proposed basic efficiency formula gave numerically equal results for the concentration of values in a concentrate and for the concentration of gangue in the tailing, for a given operation, was not proof that fundamental relationship had been described. Other formulae were possible that could also do that. For example, the formula of Luyken and Bierbrauer,† which was η_{abs}, or the formula of van Ebbenhorst, Tengbergen and Rietema. (Incidentally those could be shown to be identical formulae.)

When the components of a two-component mixture were *completely separated from each other* there was little doubt that the efficiency of separation was equal for both components, and in that respect he agreed with Mr. Douglas. The formula for concentration efficiency (*Ec*) was in accord with that view; i.e. when an operation achieved 100 per cent grade and 100 per cent recovery, the concentration efficiency was the maximum that could be attained, and was 100 per cent whichever component was considered. The denominator of the formula for concentration efficiency (*Ec*) was $h\,(1 - h)$, and that was a constant for any given mixture; it represented complete separation.

The proposed basic efficiency formula, however, was based upon the assumption that efficiency of concentration was also equal, for each component, when separations were incomplete. There was no basis for

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*Weight recovery versus value recovery.

*STEVENS, J. R., and COLLINS, D. N. New expressions for concentration efficiency and concentration index in the assessment of mineral concentration processes. U.K.A.E.A. Research Group Report R2922, Harwell, 1959. 32 p.

†LUYKEN, W., and BIERBRAUER, E. Calculations in ore dressing. *Trans. Amer. Inst. Min. Engrs*, **87**, 1930 (Milling methods), 429-51.

such an assumption. On the contrary, consideration of the energy of a two-component system led to the conclusion that only when complete separation was achieved was the useful work done on each component equal.

The technical efficiency of a concentration operation could be derived using gravitational energy as an analogue and assuming unit activity for the components of a two-component mixture.

$$\text{Efficiency} = \frac{\text{Useful WORK done}}{\text{Useful WORK}_{\text{max}}} \propto \frac{\text{MASS} \times \text{POTENTIAL DIFFERENCE}}{(\text{MASS} \times \text{POTENTIAL DIFFERENCE})_{\text{max}}}$$

$$\equiv \frac{\text{Mass of Values in Concentrate} \times \Delta[\text{Values}]}{\text{Mass of Values in Concentrate}_{\text{max}} \times \Delta[\text{Values}]_{\text{max}}}$$

$$= \frac{Cc\,.\,(c - h)}{h\,.\,(1 - h)} \text{ (as decimal fraction)}$$

or alternatively $R\,.\,\dfrac{(c - h)}{(100 - h)}$ per cent.

Thus concentration efficiency *Ec* had been derived in terms of useful work, i.e. work done on the useful component, which could of course be either component by choice. (It should be noted that for a two-component system the maximum useful work was a fixed quantity, for a given mixture.)

The author's proposed formula for 'basic efficiency':

$$E\% = \frac{(c - h)}{(100 - h)} \cdot \frac{R - C}{100 - C} \cdot 100\% \text{ (eq. 4)}$$

could be shown to equal $\dfrac{(c - h)(Cc - Ch)}{(1 - h)(h - Ch)}$ (as decimal fraction).

Therefore 'basic efficiency' $= \dfrac{\Delta[\text{Values}]\,.\,\Delta(\text{Mass of Values})}{\Delta[\text{Values}]_{\text{max}}\,.\,\Delta(\text{Mass of Values})_{\text{max}}}$.

There was surely no analogue for the author's form of efficiency expression and he suggested that the 'basic efficiency' formula was empirical while the formula for concentration efficiency (*Ec*) was 'fundamental'. More seriously still the denominator of the basic efficiency formula contained a variable; that was untenable. The denominator had different values for separations, of the same mixture, that resulted in equal grades of concentrates but differing concentrate weights. Thus it was implied that the maximum useful work that could be done in completely separating a given mixture was a variable. He felt that Mr. Douglas should look a little more closely at the implicit assumption in the bald statement given for efficiency of transfer of values on page 700, line 4.

For all the examples given in the paper, the formula for concentration efficiency, *Ec*, gave the correct evaluation in terms of useful work achieved compared to the maximum possible useful work.

Mr. D. N. Collins, before dealing with the application of basic and technical efficiency formulae, referred to the similarity between various formulae commonly used for measuring technical efficiency, and their significance. All such formulae could be presented in the form of *R* (percentage recovery of values) and *Jb* (percentage rejection of gangue). The relationships, though in different symbols, were unmistakably the same for a two-component system. Gaudin's Selectivity Index = the square root of $\dfrac{R \times Jb}{(100 - R)(100 - Jb)}$ appeared to be unnecessarily complicated in that it gave a double weighting to both recovery and rejection terms, but seemed to give more reasonable results than earlier formulae and was also the first attempt at a non-linear relationship between the two terms.

The technical efficiency was the first efficiency formula to use a non-linear relationship and basically it involved modifying the rejection term to accommodate a sampling operation—

$$Ec = \frac{R \times (Jb - (100 - C))}{(Jb_{\text{max}} - (100 - C))}$$

where C = per cent concentrate weight.

The rejection term was so constructed that any sampling operation (where $Jb = 100 - C$) would result in nil per cent efficiency and that for a perfect separation the resultant efficiency would be 100 per cent. It also had the distinct advantage that for large values of *C* and hence low grades of concentrate, the *Jb* term would be considerably reduced. Hence the efficiency term was heavily penalized for low concentrate grades and a differential was made between a good scalping operation and a good concentration operation.

$$\text{Basic efficiency } E = \frac{R - C}{R_{\text{max}} - C} \times \frac{Jb - (100 - C)}{Jb_{\text{max}} - (100 - C)}$$

The only difference between *Ec* and *E* was that in the technical efficiency formula *R* was an absolute term whereas in the basic efficiency expression it was also modified to account for material that would be recovered by a direct sampling operation. However, though a mineral dresser did not evaluate his results in terms of how much gangue material he had thrown away, but rather on the grade of the final product, he did evaluate them directly on his recovery figures, and from a practical standpoint $R \times$ rejection term was more realistic.

The present author had also pointed out, quite correctly, that the basic efficiency formula gave the same efficiency for the concentration of both valuable and gangue minerals. For that matter so did previous formulae and it was with that point in mind that he now turned to the practical applications of the two formulae.

A convenient example was that given by the author in his evaluation of cyclone products as shown in Fig. 2 and Table IV on pages 702 and 703. The speaker at that point wished to correct one figure in Table IV. The

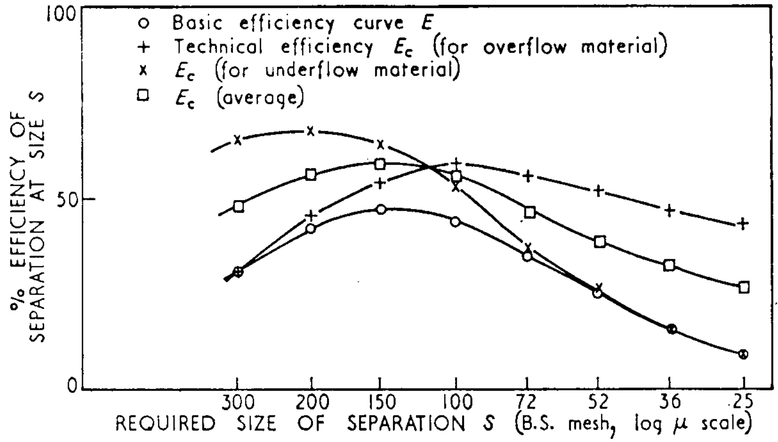

Fig. B.—Relationship between technical and basic efficiencies for a classification operation.

+300 mesh retained in the cyclone underflow should have read 97·8 and not 100 per cent as had been listed. That made a considerable difference to the *Ec* curve in Fig. 2 (see Fig. B above). The 100 per cent had been assumed, and he had obtained the 97·8 from a log-log plot of the cyclone underflow material. Furthermore, the *Ec* curve shown in Fig. 2 was an analysis of cumulative per cent coarser material and represented the efficiency of separation into the underflow. The efficiency curve for separation into the overflow, based on cumulative per cent finer material, was of a different form (see Fig. B).

He thought the optimum efficiency for separation of overflow material should be different from the optimum efficiency for separation of underflow material. Assuming that the cyclone was being used in closed circuit with a mill and that the overflow represented the feed to a separation technique, e.g. flotation, the size analysis of the overflow product could be examined.

The author had arrived at an optimum separation size of 150 mesh for that product. The analysis of that material showed that 83·7 per cent of the −150 mesh material had been recovered in a cyclone overflow product which contained 23·3 per cent +150 mesh. Thus, for a flotation feed material nominally at −150 mesh there would be present 23·3 per cent of oversize. That, he thought would be agreed, was an excessive amount of oversize. The optimum separation size using the technical efficiency was 100 mesh at which size 72·9 per cent of the −100 mesh material was recovered in the overflow product which contained only 10·4 per cent oversize. In that case, it would be seen that the overflow contained 90 per cent passing separation size, which was normally quite acceptable. The resultant loss of 10 per cent more fine material in the underflow would be taken up quite adequately in the circulating load to the mill.

Alternatively, they could consider the cyclone as preparing a coarse product for tabling where the underflow became the feed material for concentration. It was highly likely that the separation mesh was chosen because material finer than that size would not respond to that form of separation and would also affect the separation of the coarse material. Thus, the presence of nearly 20 per cent fines, which was the analysis of the +100 mesh product, was unsatisfactory.

Their optimum efficiency for that separation was at 200 mesh where 81·6 per cent of the +200 mesh material was recovered at a 95·3 per cent grade.

The basic efficiency formula appeared to be a measure of the average efficiency of the separation which incidentally could be obtained by the addition of the technical efficiency for both coarse and fine sizes. The resultant curve for that followed the contour of the curve in Fig. 2 very closely but it was of slightly higher magnitude than his (see Fig. B).

His second example was of the treatment of a glass sand. The sand in the following metallurgical balance was being used for bottle making and must not contain more than 0·25 per cent Fe_2O_3.

	% *wt.*	% *Fe_2O_3*	% *SiO_2*	% *Distribution* *Fe_2O_3*	*SiO_2*
Concentrate .	5·9	32·2	67·8	95·0	4·1
Tailing . .	94·1	0·16	99·84	5·0	95·9
	100·0	2·0	98·0	100·0	100·0

It could be seen that as a sand-cleaning operation the separation was extremely good but as an iron ore separation (always supposing one would treat an ore containing 2 per cent Fe_2O_3), the results were not comparable.

The efficiencies were:

$$\begin{aligned} &Ec \text{ (for } Fe_2O_3 \text{ separation)} &= 29\cdot 3\\ &Ec \text{ (for sand cleaning)} &= 91\cdot 1\\ &E &= 28\cdot 6. \end{aligned}$$

Mr. F. D. L. Noakes said that he would like to congratulate the author on having developed what was, as he had himself underlined, essentially a simple formula. It was on that very fact of simplicity that he wanted to take the author to task, because he had not, at the same time, issued any warnings about its use. It was so simple that the non-technical man not fully conversant with mineral dressing might think that it could be used for the direct calculation of the efficiencies of operating plants. The obvious formula to use for such a purpose would, however, appear to be the one based on chemical assays (formula 5, p. 700). Immediately, the non-expert might get into trouble because he had to work out the maximum theoretical assay of a pure concentrate (c_{max}). Quite apart from the wide variations in mineralogical composition, depending on the actual ore being mined and treated in the plant day by day, in many operations the true c_{max} could not be established by the chemical assay alone. For example, a copper ore might be predominantly chalcopyritic, but would contain varying ratios of bornite, chalcocite, etc.; or a wolfram ore might yield concentrates of similar WO_3 assays from ores of widely differing wolframite/scheelite ratios and hence different $c_{\max}$ values. An enlightened metallurgist could overcome the latter difficulty by a study of the complete analyses, deducting only gangue constituents to give a reasonable $c_{\max}$, but a less experienced man might be much misled into gaining the impression that the 'efficiency' of the plant concerned appeared to be, say, about 30 per cent.

Any formula of that nature should be used only in its proper context, namely, for comparing a number of tests carried out on similar samples. However, in spite of those, he suggested that operating plant results could be compared from day to day, or preferably from month to month, by using the formula in conjunction with a planned system of laboratory testwork. Monthly composite samples of mill feed would be batch tested by a standardized method as closely parallel to the plant operating conditions as possible. The same or precisely similar samples would then be used for a series of optimizing batch tests to establish the maximum 'basic efficiency' obtainable with the ore, the calculations being based on the maximum concentrate grade by assay or mineral count. By working back through the standard laboratory tests to the actual results obtained in the plant, suitable factors could be developed whereby any variations in performance would show up and, frequently, an indication would be obtained as to whether the variations were due to changes in the ore feed or to faulty plant operation. Although that might seem to be a laborious procedure, it was his opinion that any short cut would only lead to wrong conclusions being reached and certainly any direct application of the 'basic efficiency formula' to plant results would be completely misleading.

The speaker had used the expression 'basic efficiency' during the development of his argument, but he was not in favour of the use of the word 'efficiency' in that context for the reasons outlined. It was his opinion that the word 'efficiency' was very much overworked in many branches of technology and industry. He suggested that a far less misleading impression would be created if the fractions in the formulae developed in the paper were not multiplied by 100 and that the functions derived should be termed 'basic performance factors' or 'indices'.

In spite of Dr. Robinson's earlier remarks, the efficiency that should basically be of interest was the efficiency of converting the muck that the miners produced into money for the shareholders' benefit. Any real efficiency formula would introduce such factors as were sometimes ignored by the mill operators themselves—in particular, the actual value of the concentrates. Generally, the mill man was made very conscious of his operating costs, but he was very often quite unaware of the effect of variations in the composition of his products on their actual value as revenue to the company, which involved such factors as freight, insurance, smelter contract terms and so on, or, on the bigger mines with smelters, etc., factors related to the recovery and sale of the final metal. For a useful overall efficiency formula he felt all those factors would have to be incorporated. Fortunately, that would involve so many different 'variable constants' that it would need a metallurgist or mining engineer to interpret it. That being so, it would generally seem preferable to provide the operating staff with the essential financial data, enabling them to calculate their economic efficiencies on the basis of their knowledge of the technology. Any attempt to take a short cut by making some sort of overall efficiency slide-rule, suitable for the non-technical user, seemed doomed to failure.

Mr. C. C. Dell said that one of the fundamental problems of mineral dressing was reconciling quantity with quality, i.e. the recovery with grade. That problem made itself felt every time a test was carried out, whether the test was on a laboratory scale or full scale in the plant. Whatever the test, an unambiguous result could only be obtained if all the relevant information were condensed into a single criterion of efficiency. Many formulae and graphical studies had been put forward for solving that problem, the new one being, it was claimed, more logical than the others. The requirements which were set out in the first two pages of the paper were by way of being a statement of the premises from which the basic efficiency formula was then derived. The speaker's criticism was not on the logic of the derivation of the formula but on the original requirements on which it was based. He suggested that an efficiency formula should satisfy two requirements only:

(1) It should be directly proportional to the economic gain from the separation, being zero when there was neither gain nor loss and 100 for maximum possible gain.

(2) The expression should be dimensionless.

It might be argued that to bring economics into account would inevitably make the criterion of efficiency sensitive to daily market fluctuations. That was not the case, however, as it was only the relative economic importance of the grade and recovery which had to be assessed, and that did not fluctuate with market conditions as did the absolute price of a valuable mineral. If the relative importance of the grade and recovery were fixed implicitly, as in the basic efficiency formula, then too much emphasis would be placed on recovery in an example such as fluorspar beneficiation and too much emphasis on grade in an example such as the concentration of galena. If it were impossible to avoid such a compromise, then such criticisms would have little point. However, an economic, dimensionless efficiency expression did exist,* and he would be interested to hear the author's reasons for ruling out economics at the start of his argument.

He would criticize the basic efficiency formula in yet another respect: that it took no account of the rate of separation. Rate of separation was clearly the factor which influenced the size of machine, or the number of machines required to treat a given rate of throughput. If rate were not taken

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*Dell, C. C. The analysis of flotation test data. *Colo. Sch. Mines Quart.*, **56**, no. 3, July 1961, 113–27. (50th Anniversary of Froth Flotation in the U.S.A.)

——. The evaluation of flotation performance. *Colo. Sch. Mines Quart.*, **56**, no. 3, July 1961, 129–40. (50th Anniversary of Froth Flotation in the U.S.A.)

——. A comprehensive criterion of coal-cleaning efficiency. *J. Inst. Fuel*, **35**, June 1962, 240–5.